Deep Sea Duel for iOS, Android, and Your Desktop
Strategy Guide

Using Deep Sea Duel in Pre-K–Grade 12

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Deep Sea Duel Strategy Guide: Using Deep Sea Duel in Pre-K−Grade 12.

The National Council of Teachers of Mathematics is the public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research.

Introduction

This strategy guide provides a basis to the explanation of the strategy behind Deep Sea Duel. It includes a discussion of the isomorphic relationship between Deep Sea Duel and tic-tac-toe, along with an explanation of the means to construct a magic square. For the rules and instructions, please refer to *Deep Sea Duel Instructional Guide: Using Deep Sea Duel in Pre-K−Grade 12.*

- **History of the Game**
  the game is based on a journal article
- **The Strategy**
  an explanation of how not to lose
- **Questions for Students, with Teacher Answers**
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History of the Game

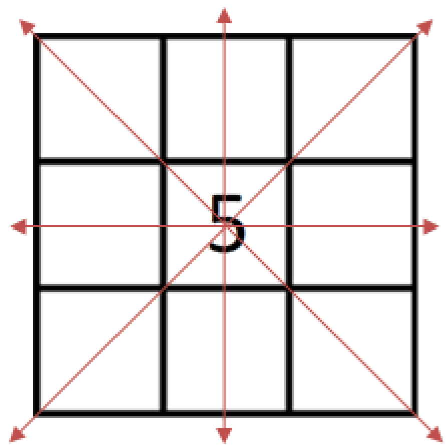
This game is based on the article, "What Is the Name of This Game?" by John Mahoney, which appeared in the October 2005 issue of *Mathematics Teaching in the Middle School*, vol. 11, no. 3, pp. 150–154.

The Strategy

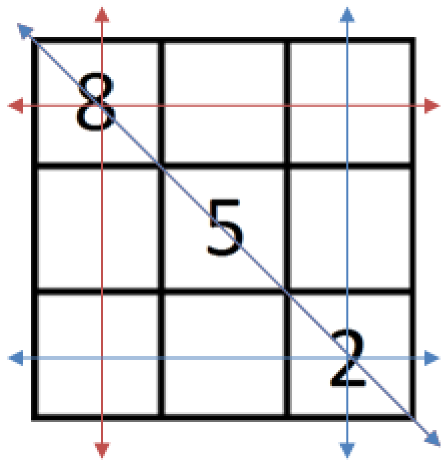
An *isomorphism* is a situation where two things behave the same and have the same rules but look different. The 9-bubble game is an isomorphism to a very simple two player game: tic-tac-toe! Let's see why that is. In both games, you play against one opponent. You need three in a row to win in tic-tac-toe, and you need three in the correct combination to win in Deep Sea Duel. There are nine potential moves in each game, and once a space or number is taken it cannot be replayed. In tic-tac-toe, you can block your opponent from winning by taking a space that would complete his or her line, and in Deep Sea Duel you can block an opponent by taking a bubble that would complete a combination. Each tic-tac-toe space is unique, and the strategy of play depends on who starts; likewise, each bubble is unique, and the strategy depends on how Deep Sea Duel is started. Some moves are more advantageous than others and have a higher probability of success. The question remains: How do these similarities help with Deep Sea Duel? Can the bubbles be arranged in a tic-tac-toe board? The answer is yes, in something called a magic square. A magic square is an arrangement of the numbers 1, 2, …, n^2 in an $n \times n$ matrix such that the sum of the numbers in every row, column, and diagonal is the same number. Each number may only be used once. Further discussion of the origins and history of magic squares can be found in the final section of this guide.

How to Create a 3×3 Magic Square
First, take all the positive integers 1 through 9, and write down all the combinations of three numbers that will yield a sum of 15 (a proof of why the sum must be 15 is shown in the Questions for Students section). How many times is each number used to get a winning combination? For example, 8 is used in {3, 4, 8}, {1, 6, 8}, and {2, 5, 8}, so the number 8 is used in three different combinations. Which numbers are used more frequently? Since we know we are trying to fill the spaces in a tic-tac-toe board, where should the numbers that occur more frequently go?

Because we know 5 can be used in four different combinations, the most frequent number to appear in winning combinations, it should appear in the middle of the board - the only location in the grid that allows for a win in four different ways.



Now we also see that 2, 4, 6, and 8 are used three times in combinations. Since the corners of the tic-tac-toe board have the potential to be a part of three different winning outcomes, we can see that 2, 4, 6, and 8 must be located in the corners of the board. Also observe that in relation to the existing 5 on the board, {2, **5**, 8} and {4, **5**, 6} are winning combinations. So it seems that the 2 and 8 should be diagonal from each other.



Similarly, 4 and 6 should be on the other diagonal of the board.

8		6
	5	
4		2

Use the remaining numbers and simple algebra to find out where the rest of the numbers must go. For example, the number in the middle of the top row can be found by solving the equation $8 + x + 6 = 15$. Since $x = 1$, 1 must be placed in the middle of the top row. This is good practice in algebra and arithmetic. The result of this should look like this:

8	1	6
3	5	7
4	9	2

Note that if your magic square looks different, it may still be correct; a magic square can be rotated or flipped without changing its properties. Observe how it is preserved through a vertical flip and a 180-degree turn about the center:

2	9	4
7	5	3
6	1	8

6	1	8
7	5	3
2	9	4

Alternative Modes and 16 Bubbles

If you go beyond using the numbers 1 through 9 in the 9-bubble game, the same principle holds. The difference is that each bubble has been changed, but the transformation is linear, meaning that each number in the original set, {1, 2, 3, 4, 5, 6, 7, 8, 9}, has been transformed in the exact same way. Look at the following game:

The bubbles in sequential order are: {-4.1, -2.2, -0.3, 1.6, 3.5, 5.4, 7.3, 9.2, and 11.1}. In other words, 1 has become -4.1, 2 has become -2.2, and so on. We know that these values can be put into a magic square because linear transformation has occurred. How do we know? The answer can be found by solving a system of linear equations:

$1x + y = -4.1$ and $2x + y = -2.2$; in other words, what happened to one in order to make it -4.1, and what happened to 2 in order for it to become -2.2? Solving this system gives you that $x = 1.9$ and $y = -6$. Thus, all the numbers 1, 2, 3, …, 9 have been multiplied by 1.9 first and then subtracted by 6. Testing this with all the values from 1 through 9 prove that a linear transformation has indeed occurred. The new magic square is shown below. Notice that every column, row, and diagonal has a sum of 10.5.

8	1	6
3	5	7
4	9	2

9.2	-4.1	5.4
-0.3	3.5	7.3
1.6	11	-2.2

The only thing left to do now is play tic-tac-toe against a friend or Okta. Examples are shown in the Questions for Students section.

Similarly, the 16-bubble game can be likened to a 4x4 tic-tac-toe board. The construction of a 4x4 magic square is left as an exercise to the reader. A quick Internet search will provide 4x4 magic squares and algorithms for their construction.

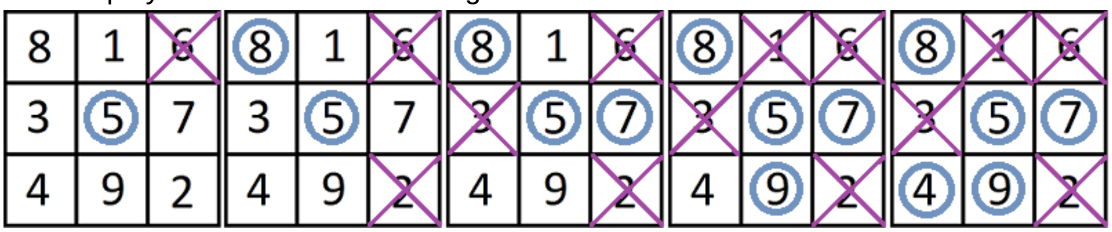
A No-Lose Strategy
Thus, the card game has been reduced to a tic-tac-toe board. Since there is no way to ensure a win in tic-tac-toe, there is also no way to guarantee that you will always win in Deep Sea Duel. Assuming that both players play with optimal strategy, each person can only ensure that they do not lose. Seeing Deep Sea Duel like a tic-tac-toe grid makes it easier to play defensively and easier to recognize when someone has won. Once a player gets three in a row in the magic square tic-tac-toe, they have won the game.

There are existing tic-tac-toe strategy guides that can guarantee a win or a tie game regardless of who starts and what moves they use. A quick Internet search should yield a plethora of ready-made resources. Refer to the next section for concrete examples.

Questions for Students, with Teacher Answers

1. How many ways can you make 15 using 3 of the 9 bubbles? What are they?
   [8: {1, 6, 8}, {1, 5, 9}, {2, 6, 7}, {2, 4, 9}, {2, 5, 8}, {3, 4, 8}, {3, 5, 7}, {4, 5, 6}]
2. What do you think the best number to use is, and why?
   [5, because it occurs in the most combinations. Or 2, 4, 6, 8, because optimal tic-tac-toe strategy is to start in a corner.]
3. Will someone always win? Prove it!
   [No. There's a Tie option for the outcome of the game.
   For example:
   Round 1: Player 1 chose 5, and Player 2 chose 6
   P1: {5}

P2: {6}
Round 2: Player 1 chose 8, and Player 2 chose 2 to block P1 from getting 15
P1: {5, 8}
P2: {6, 2}
Round 3: Player 1 chose 7, andPlayer 2 chose 3 to block P1 from getting 15
P1: {5, 8, 7}
P2: {6, 2, 3}
Round 4: Player 1 chose 9, and Player 2 chose 1 to block P1 from getting 15
P1: {5, 8, 7, 9}
P2: {6, 2, 3, 1}
Round 5: Player 1 chose 4, and there are no bubbles remaining.
Neither player has a sum of 15 using a combination of 3 bubbles.



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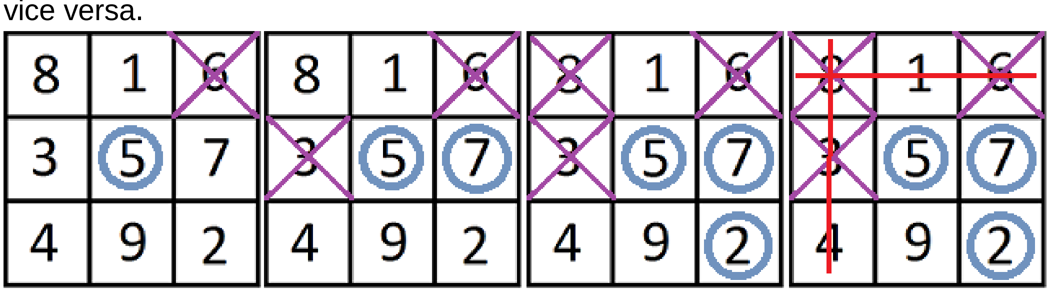
4. Is it better to go first or second?

[Neither is better, as both players are guaranteed a win or tie if they play strategically.
For example, Player 2 could win like this:
Round 1: Player 1 chose 5, and Player 2 chose 6
P1: {5}
P2: {6}
Round 2: Player 1 chose 7, and Player 2 chose 3 to block Player 1 from getting 15.
P1: {5, 7}
P2: {6, 3}
Round 3: Player 1 chose 2, and Player 2 chose 8 to block Player 1 from getting 15.
P1: {5, 7, 2}
P2: {6, 3, 8}
Now, Player 2 is guaranteed a win. If Player 1 chose 4, Player 2 would choose 1 and vice versa.



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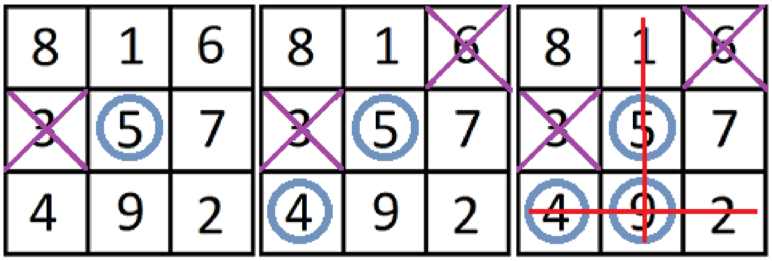
5. Suppose you went first. The target sum is 15 with the numbers 1 through 9 to choose from. Suppose you pick a 5, and your partner chooses 3. What should you choose next so that you are GUARANTEED a win?

[To guarantee a win, choose 4 or 8.

Round 1: Player 1 chose 5, and Player 2 chose 3
P1: {5}
P2: {3}
Round 2: Player 1 chose 4, and Player 2 chose 6 to block P1 from getting 15
P1: {5, 4}
P2: {3, 6}
Round 3: Player 1 chose 9, and Player 2 **cannot win.**
If P2 takes 2 then P1 will take 1 and vice versa. P1 is guaranteed a win
P1: {5, 4, 9}
P2: {3, 6}



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6. Consider another scenario. Suppose you choose a 5 first. What should your opponent choose to stop your guaranteed win?

[2, 4, 6 or 8.]

7. Is there an optimal strategy?

[Yes. As the first player, start with an even number.]

8. Why is 15 the target sum for the bubbles 1 through 9? Why is 34 the target sum for the bubbles 1 through 16?

[For the 9-bubble game, a 3×3 magic square is needed. Assuming that we do not know the placement of the numbers, let's label them *a*, *b*, …, *i*:

a	b	c
d	e	f
g	h	i

We know that $a + b + \ldots + i = 45$, the sum of all the numbers from 1 through 9. Using the associative property of equality, we find that $(a + b + c) + (d + e + f) + (g + h + i) = 45$. Since every row must add up to the same value, $(a + b + c) = (d + e + f) = (g + h + i)$; let's call that value *s*. Then, $s + s + s$, or $3s$ is equal to 45. This means that *s*, or the value of the sum of every row, must be 15. A similar proof is used to show that the target sum for the bubbles 1 through 16 is 34.]

9. Generate a formula to find out what the target value should be of any given 9 or 16 values.

[By using the same magic square above, we know that $a + b + \ldots + i = S$, where *S* is the new sum. Using the associative property of equality, we find that $(a + b + c) + (d + e + f) + (g + h + i) = S$. Since every row must add up to the same value, $(a + b + c) = (d + e + f) = (g + h + i)$; let's call that value *s*. Then, $s + s + s$, or $3s$ is equal to *S*. Thus, the target value, *s*, is $S/3$. In other words, the target value can be found by adding up all the nine numbers, and then dividing the sum by 3. A similar proof will show that the target value for any given 16 bubbles should be

S/4.

Throughout playing, continue to ask students to think about what game this resembles.

A History of Magic Squares

Magic squares have been around for thousands of years. The earliest recorded appearance of magic squares was in Ancient China around 2200 B.C. According to legend the magic square was on a turtle shell of a river god during a flood, and the sum of 15 was significant because in the Chinese calendar there were 15 days in each of the 24 months. The magic square also makes an appearance in Indian, Arab, Egyptian and medieval European art. Magic squares were believed to have special properties such as long life and divination so they were made into talismans. Even Benjamin Franklin made magic squares, one 8x8 and another especially magical 16x16 that has rows and columns with the same sum as well as bent diagonals and all 4x4 squares inside the 16x16 magic square. Today, there is still a fascination with the magic square; it has even extended to three dimensions, namely magic cubes.

Alignment to NCTM Standards and the Common Core

The rules and strategy behind Deep Sea Duel relate to a number of standards and practices from NCTM's *Principles and Standards for School Mathematics* (NCTM 2000) and from the Common Core State Standards for Mathematics (NGA Center and CCSSO 2010). These are listed below.

NCTM's *Principles and Standards*

- Pre-K–2: Number and Operations
  - Understand numbers, ways of representing numbers, relationships among numbers, and number systems:
    - develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers.
  - Understand meanings of operations and how they relate to one another:
    - understand the effects of adding and subtracting whole numbers.
  - Compute fluently and make reasonable estimates:
    - develop and use strategies for whole-number computations, with a focus on addition and subtraction
    - develop fluency with basic number combinations for addition and subtraction
    - use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators.
- Grades 3–5: Number and Operations
  - Understand numbers, ways of representing numbers, relationships among numbers, and number systems:
    - understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals
  - Compute fluently and make reasonable estimates:

        - develop fluency in adding, subtracting, multiplying, and dividing whole numbers
        - select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tool.
- Grades 6–8: Algebra
    - Represent and analyze mathematical situations and structures using algebraic symbols
        - develop an initial conceptual understanding of different uses of variables
        - use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships
        - recognize and generate equivalent forms for simple algebraic expressions and solve linear equations..
- Grades 9–12: Number and Operations
    - Understand numbers, ways of representing numbers, relationships among numbers, and number systems:
        - use number-theory arguments to justify relationships involving whole numbers.

Common Core State Standards for Mathematics (CCSSM)

- Kindergarten
    - Counting and Cardinality (K.CC)
        - Compare numbers.
            - Compare two numbers between 1 and 10 presented as written numerals. (K.CC.7)
    - Operations and Algebraic Thinking (K.OA)
        - Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
            - Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (K.OA.1)
            - For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. (K.OA.4)
    - Numbers and Operations in Base Ten (K.NBT)
        - Work with numbers 11–19 to gain foundations for place value.
            - Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (K.NBT.1)
- Grade 1

- Operations and Algebraic Thinking (1.OA)
  - Understand and apply properties of operations and the relationship between addition and subtraction.
    - Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) (1.OA.3)
    - Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. (1.OA.4)
  - Add and subtract within 20.
    - Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). (1.OA.5)
    - Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (1.OA.6)
  - Work with addition and subtraction equations.
    - Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. (1.OA.7)
    - Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, 6 + 6 = ?. (1.OA.8)
- Number and Operations in Base Ten (1.NBT)
  - Use place value understanding and properties of operations to add and subtract.
    - Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. (1.NBT.4)

- Grade 2
  - Operations and Algebraic Thinking (2.OA)

        - Add and subtract within 20.
            - Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers (2.OA.2)
    - Numbers and Operations in Base Ten (2.NBT)
        - Use place value understanding and properties of operations to add and subtract.
            - Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (2.OA.5)
            - Add up to four two-digit numbers using strategies based on place value and properties of operations. (2.OA.6)
            - Explain why addition and subtraction strategies work, using place value and the properties of operations. (2.OA.9)
- Grade 3
    - Operations and Algebraic Thinking (3.OA)
        - Solve problems involving the four operations, and identify and explain patterns in arithmetic.
            - Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. (3.OA.9)
    - Numbers and Operations in Base Ten (3.NBT)
        - Use place value understanding and properties of operations to perform multi-digit arithmetic.
            - Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (3.NBT.2)
- Grade 5
    - Number and Operations in Base Ten (5.NBT)
        - Perform operations with multi-digit whole numbers and with decimals to hundredths.
            - Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (5.NBT.7)
- Grade 6
    - Expressions and Equations (6.EE)
        - Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true (6.EE.5)

- Grade 7
    - The Number System (7.NS)
        - Apply properties of operations to add and subtract rational numbers (7.NS.1d)

CCSSM Standards for Mathematical Practice (SMP)

- **Make sense of problems and persevere in solving them. (SMP.1)**
    - In order to develop a strategy to play this game, multiple mathematical jumps need to be made. First, students must observe that there are multiple combinations of numbers that will achieve the desired sum. Next, they must devise a magic square in order to see clearly that all the winning combinations can fit together in an organized fashion. Then, the magic square must be connected to the game of tic-tac-toe. Finally, a winning strategy of tic-tac-toe must be developed.
- **Construct viable arguments and critique the reasoning of others. (SMP.3)**
    - Discussion of how not to lose the game or the students' strategies in winning would require students to have an argument for why their strategy works. In addition, it would require them to discuss with other students why their strategy does or does not hold.
- **Look for and make use of structure. (SMP.7)**
    - Students can recognize that patterns exist in the winning combinations. For example, there are more combinations including 5 than any other number. This is because there are four ways of making 10 with the other numbers. Students who can recognize this pattern will need to extend it and apply this knowledge to the construction of a magic square.

Citations

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